

KUG1C3

Dasar Algoritma dan Pemrograman



Recursive Algorithm

What is recursion?

- ▶ Sometimes, the best way to solve a problem is by solving a smaller version of the exact same problem first
- ▶ Try to tear a sheet of paper into the same 8 pieces



Tear paper into the same 8 pieces

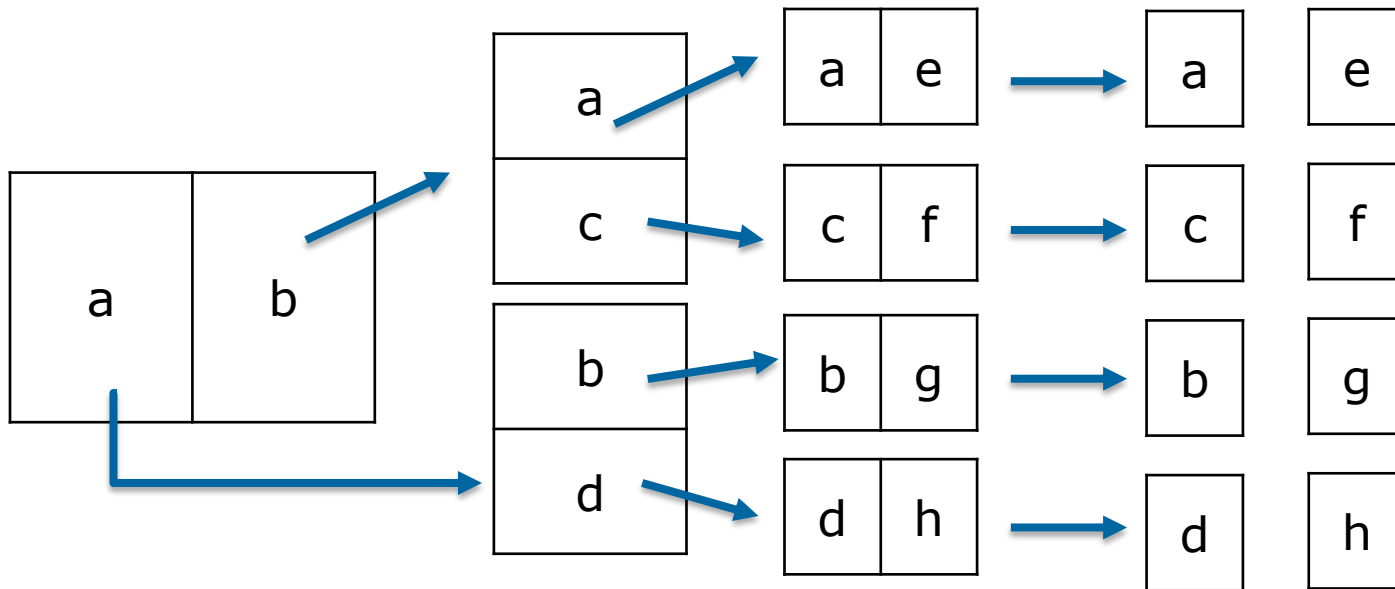
- ▶ To solve this, we can just tear it 7 times as follows:

1	2	3	4	5	6	7	8
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- ▶ That was an example of the application of looping

Tear paper into the same 8 pieces

- Or we can tear it into 2, and repeat the process for each pieces 2 times



- That is an example of the application of recursive

Some Definitions

- ▶ Recursion is a technique that solves a problem by solving a smaller problem of the same type
- ▶ Recursion is a principle closely related to mathematical induction.

- ▶ $F(0) = 0$

- ▶ $F(x) = F(x-1) + 2$

$$F(x) = \begin{cases} 0 \\ F(x-1) + 2 \end{cases}$$

Example

▶ Power of two

▶ $2^n = 2 * 2^{n-1}$

▶ $2^0 = 1$

▶ Factorial

▶ $X! = X * (X-1)!$

▶ $1! = 1$

Careful when writing

- ▶ If we use iteration, we must be careful not to create an infinite loop by accident:

```
While ( result > 0 ) do  
    result ++
```



Oops!

Careful when writing

- ▶ Similarly, if we use recursion we must be careful not to create an infinite chain of function calls

Remember the Rule!

- ▶ An Algorithm must stop!
- ▶ Define a rule that will stop the recursion
(initial set / base case)
 - $X! = X * (X-1)!$
 - $0! = 1$
- ▶ Define a rule to reach the next iteration
(construct new element / step)



Algorithm of the factorial function

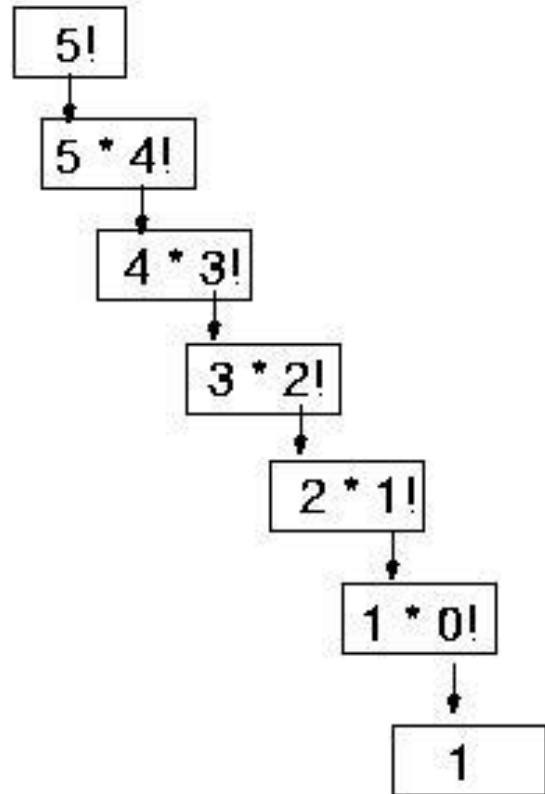
Function Factorial(input : n : integer)

if (n=0) then // **base case**

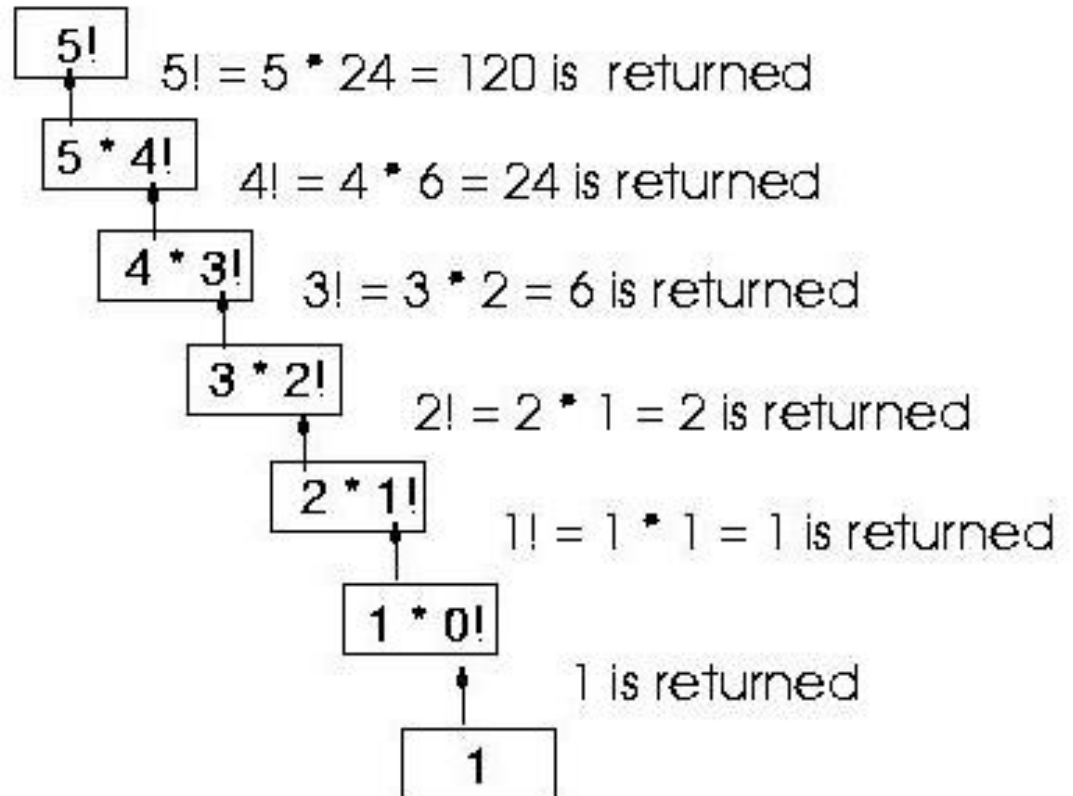
→ 1

else

→ $n * \text{Factorial}(n-1)$



Final value = 120



Recursively Defined Functions

› A famous example: The Fibonacci numbers

› $f(0) = 0, f(1) = 1$

› $f(n) = f(n - 1) + f(n - 2)$

› $f(0) = 0$

› $f(1) = 1$

› $f(2) = f(1) + f(0) = 1 + 0 = 1$

› $f(3) = f(2) + f(1) = 1 + 1 = 2$

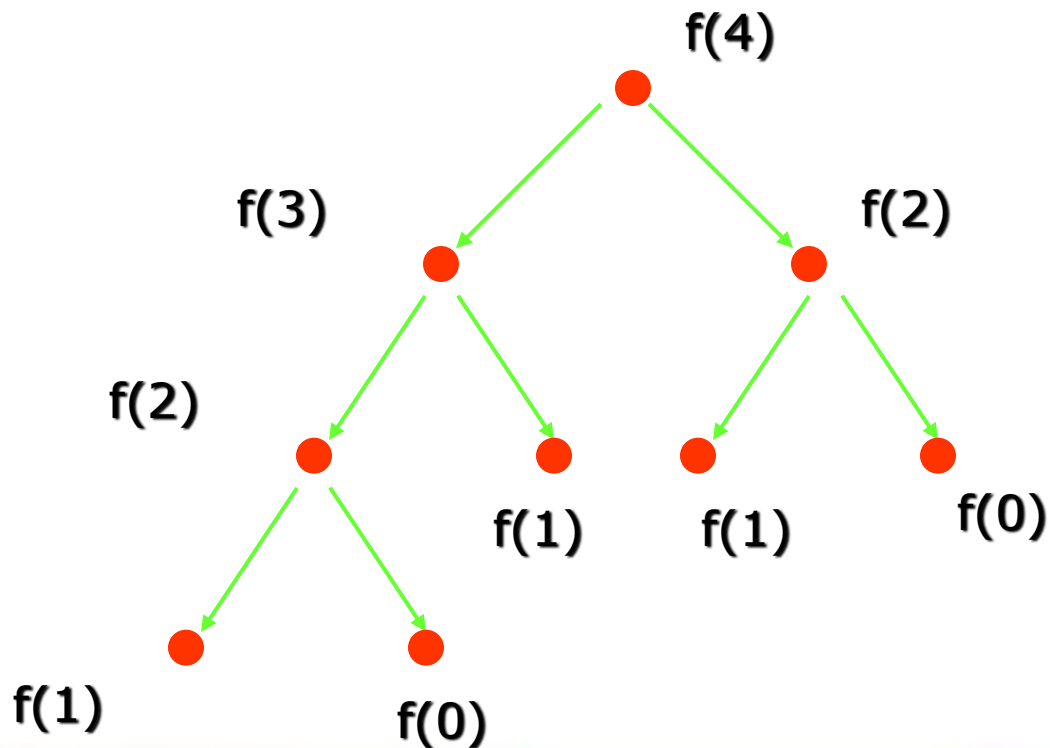
› $f(4) = f(3) + f(2) = 2 + 1 = 3$

› $f(5) = f(4) + f(3) = 3 + 2 = 5$

› $f(6) = f(5) + f(4) = 5 + 3 = 8$

Recursive Algorithms

▶ Recursive Fibonacci Evaluation:



Recursive Algorithm

- ▶ An algorithm is called recursive if it solves a problem by reducing it to an instance of the same problem with smaller input
- ▶ A recursive function must contain at least one non-recursive branch.
- ▶ The recursive calls must eventually lead to a non-recursive branch

Recursion vs. iteration

- ▶ For every recursive algorithm, there is an equivalent iterative algorithm
- ▶ Iteration can be used in place of recursion
 - An iterative algorithm uses a *looping construct*
 - A recursive algorithm uses a *branching structure*

Recursion vs. iteration

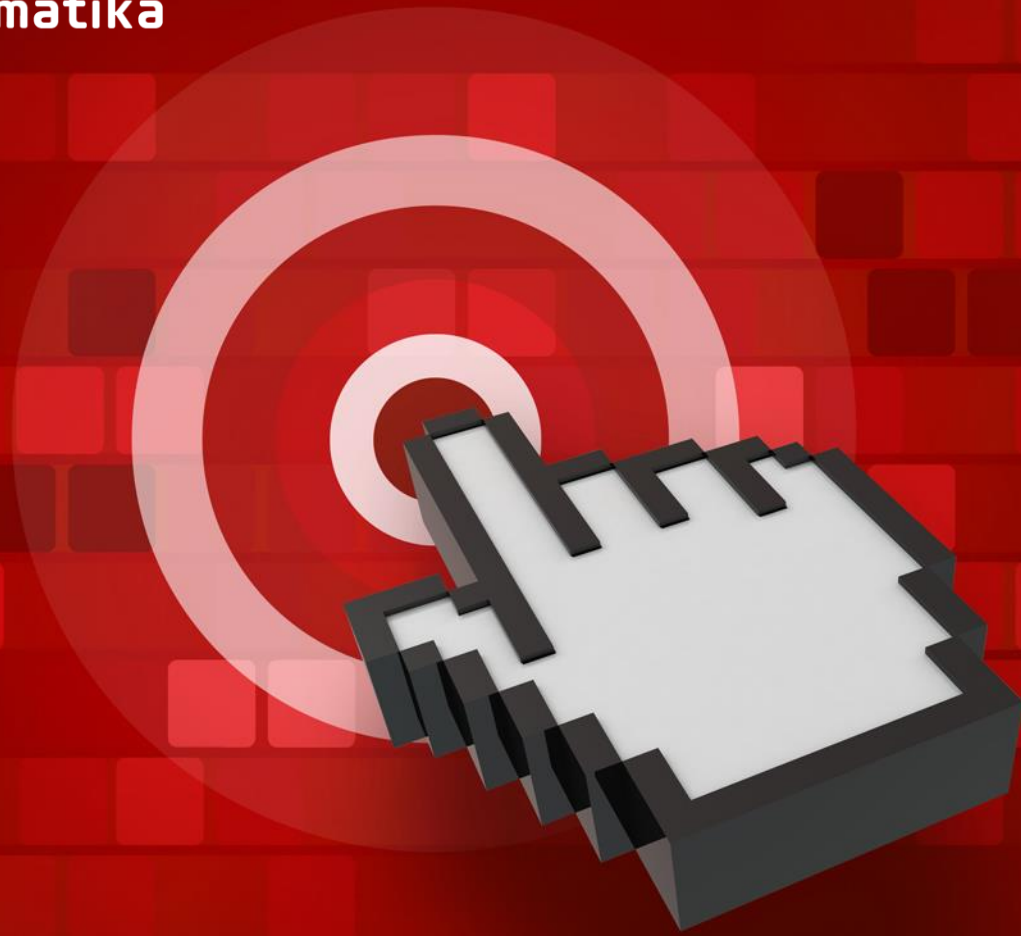
- ▶ Recursive solutions are often less efficient, in terms of both *time* and *space*, than iterative solutions
- ▶ Recursion can simplify the solution of a problem, often resulting in *shorter*, more easily understood source code

How do I write a recursive function?

- ▶ Determine the size factor
- ▶ Determine the base case(s)
(the one for which you know the answer)
- ▶ Determine the general case(s)
(the one where the problem is expressed as a smaller version of itself)
- ▶ Verify the algorithm
(use the "Three-Question-Method")

Three-Question Verification Method

- ▶ **The Base-Case Question:**
 - Is there a non-recursive way out of the function, and does the routine work correctly for this "base" case?
- ▶ **The Smaller-Caller Question:**
 - Does each recursive call to the function involve a smaller case of the original problem, leading inescapably to the base case?
- ▶ **The General-Case Question:**
 - Assuming that the recursive call(s) work correctly, does the whole function work correctly?



THANK YOU